CONSEQUENCES OF ACOUSTIC EMISSION ON CRACK SPEED AND ROUGHNESS EXPONENT IN BRITTLE DYNAMIC FRACTURE

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ABSTRACT

We show by computer simulations that acoustic emission from the crack tip strongly reduces the delivery of fracture work, due to the coupling between the crack speed and the acoustic branches in dispersive media. The direct consequence is a selection criterion for the terminal crack speed which, for planar cracks, produces results corresponding to those found in experiments on highly anisotropic materials. In case of isotropic material with cracks of unrestricted geometry, the drop in the crack speed with respect to the planar case is connected to a mechanism of attempted branching, which is also responsible for the logarithmic roughness of the final fracture for marginal loadings. Higher loadings lead to a well defined roughness exponent of $\zeta \sim 0.45$ compatible with that measured experimentally at short length scales, and in our simulations clearly connected with the generation of macroscopic branches.

1 INTRODUCTION

Acoustic emission is commonly used to analyze the precursors of fracture, exploiting the possibility to determine the location, frequency and energy of microfracturing events. Less common is the analysis of acoustic emission from moving cracks because locating a crack and measuring the energy involved is better achieved by different methods. An interesting aspect of the acoustic emission in dynamic fracture is that this phenomenon is only partially covered by continuum theory. Continuous media support the propagation of waves travelling at the characteristic sound speeds v_l and v_t of the longitudinal and transversal acoustic modes, and at the Rayleigh speed v_R for surface waves. The prediction of the continuum theory for the terminal crack speed is that a crack never exceeds v_R [1]. Up to that speed, it describes an advancing crack as a continuous deformation of the medium and no sound emission is expected because there can be no coupling between the crack speed and sound waves. Experiments in both real materials and simulations nevertheless show intense acoustic emission [2, 3], which cannot be directly accounted for in the continuum limit. This emission can be understood as a consequence of discreteness which is always present in real materials (and in simulations) and is masked in the theory by the continuum approach [4]. In the following we show that such acoustic emission influences the amount of energy which goes into fracture work depending on the crack speed. This leads to a speed selection criterion for crack propagation and influences the scaling of the final surface.

2 DISCRETENESS AND ENERGY RELEASE RATE

In presence of discreteness and hence of acoustic emission, the macroscopic energy release rate

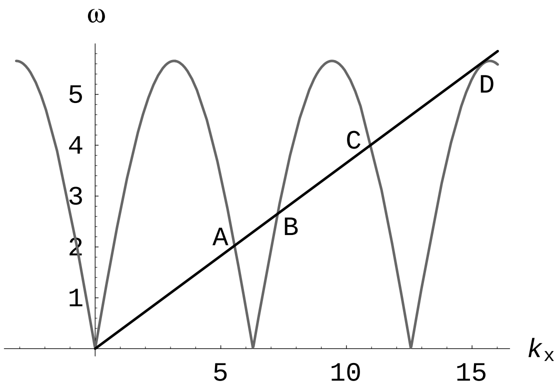

Figure 1: Schematic representation of the basic phenomenon. A periodic lattice generates a periodic acoustic band structure. A crack advancing at some speed v matches such periodic band structure at points A, B, C, and D in the example shown, emitting waves at the corresponding frequencies ω and wavevectors k_x. Resonant emission is found in D: in this case the crack speed v also matches the group velocity $\partial\omega/\partial k_x$ of emitted waves.

provided by the continuum theory corresponds to the sum of two distinct contributions [4]:

$$G_M(v,t) = G_{\mathrm{br}}(v,t) + G_{\mathrm{ph}}(v,t). \tag{1}$$

Here $G_M(v,t)$ is the solution of the continuum limit which governs the delivery of energy towards the crack tip and hence represents the energy *available* at the crack tip at a given time t. $G_{\mathrm{br}}(v,t)$ is the actual *breakage* energy release rate, or the portion of the available energy which effectively goes locally into fracture work, whilst $G_{\mathrm{ph}}(v,t)$, the *phonon* energy release rate, is the portion of the available energy which is radiated as acoustic emission. $G_M(v,t)$ can be obtained from the continuum theory for a given (computer) experimental setup, whilst $G_{\mathrm{br}}(v,t)$ can be measured in simulations by fixing the crack speed and measuring the resulting fracture work.

We have introduced a novel finite element model which permits fast three-dimensional simulations and is amenable of explicit analytical treatement [4]. The scheme adopted introduces a discretization of the continuum elastodynamic equations by using an fcc lattice of massive sites. The lattice geometry is then reflected in the acoustic properties of the material in that the materials is dispersive, and influences the behaviour of the energy release rate.

By a series of computer simulations we measured the breakage energy release rate $G_{\mathrm{br}}(v,t)$ for a planar crack, in the special case of a strip geometry with fixed displacements at the top and bottom boundaries. In this case the total available macroscopic fracture energy $G_M(v,t)$ is independent of the crack speed [1] and, for cracks longer than the height of the sample, all energy release rates of eq. (1) are time independent, so that we get $G_M^\infty = G_{\mathrm{br}}(v) + G_{\mathrm{ph}}(v)$. Although the macroscopic energy release rate is independent of the crack speed, our simulations show that the breakage energy release rate does depend on the crack speed. Such speed dependence is a consequence of the coupling between the crack speed and the acoustic emission via the acoustic dispersion relations. Figure 1 describes the basic phenomenon. At any crack speed $v < v_R$ the moving tip emits sound waves at the frequency and wavevector of the corresponding permitted acoustic modes. In particular, for some special speeds we have resonant emission when the emitted waves travel at the same speed as the crack itself, leading to an increase of the energy radiated into phonons. This in turn leads

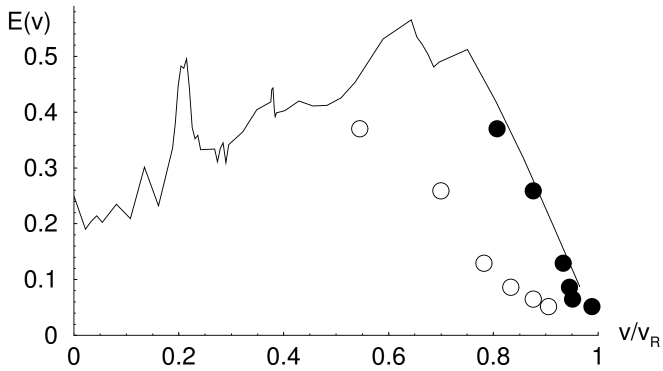

Figure 2: The continuous line shows the speed dependence of efficiency. Filled circles correspond to simulations of planar crack in a disordered medium for different loadings. The topmost point corresponds to the lowest loading and vice-versa: increasing the loading leads to an increase in the terminal crack speed. Empty circles correspond to simulations of non-planar cracks in a disordered medium; the reduced crack speed is associated with crack branching.

to a decrease in the energy going into fracture work and thus explains the speed dependence of the breakage energy release rate.

The relationship between the microscopic breakage energy release rate and the macroscopic energy release rate is better expressed by introducing the *efficiency* $E(v)$:

$$G_{\rm br}(v,t) = E(v)G_M(v,t) \tag{2}$$

In the fixed grip setup discussed above, this gives $G_{\rm br}(v) = E(v)G_M^\infty$, so that the efficiency becomes the sole source of velocity dependence. We have shown [4] that $E(v)$ only depends on the lattice geometry and crack speed, being local to the crack tip and thus independent on the macroscopic dynamical regime which instead is generally described by $G_M(v,t)$.

3 CRACK SPEED AND ROUGHNESS EXPONENT

The translation of the standard Griffith criterion in the framework of eq. (1) is that a crack will advance only when $G_{\rm br}(v,t)$ exceeds a threshold value connected with the toughness of the material. In the fixed grip setup discussed above, this translates into a threshold for the efficiency $E(v)$. For a given threshold value for $E(v)$ the possible crack speeds are directly obtained by looking at its speed dependence in fig. 2, and simple stability argument [4] leads to the conclusion that stable crack propagation is only possible when $E(v)$ is a decreasing function of the crack speed.

We have performed numerical simulations of moving cracks by fixing the threshold and varying the applied loading. In figure 2 full circles correspond to simulations of planar cracks in a disordered medium for different loadings: all points fall on the efficiency curve. Furthermore, for low loadings (corresponding in the figure to high values of $E(v)$) the crack speed is fully compatible with the value measured in highly anisotropic materials [5, 6]. Empty circles show the results on the terminal crack speed for nonplanar cracks, when out of plane breakage is allowed. We have verified [7] that the drop in crack speed, with respect to the planar case, is directly connected with a mechanism of *attempted branching*, when new branches try to open. In this situation, not all the energy is used to have the crack advance, and the crack slows down. On the other hand, the energy available is insufficient to initiate macroscopic branches, and the resulting topology is compatible with a logarithmic scaling,

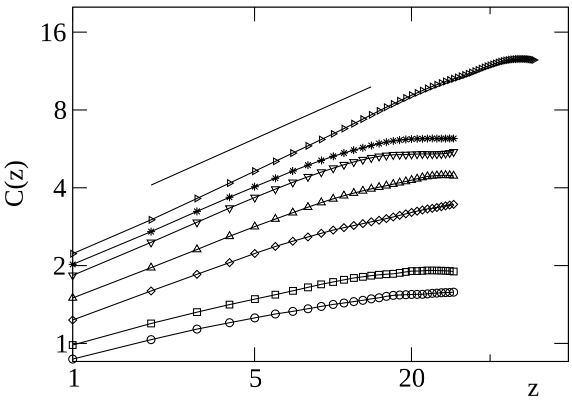

Figure 3: The height-height correlation function of the final fracture surface for increasing loadings. The highest curves have a slope similar to the reference slope $\zeta = 0.45$ shown by the continuous line. The lowest curves correspond to the lowest loadings: note that they deviate from the reference slope. It can be shown that for such loadings the scaling of the surface is logarithmic [4].

found in experiments of marginal loading [8] and in theoretical calculations [9, 10]. Only for higher loadings does the roughness exponent ζ grow to values of $\zeta \sim 0.45$ (see fig. 3) found in many experiments [11, 12, 13], and this is connected with the appearence of macroscopic branches in our simulations.

4 CONCLUSIONS

We have shown that acoustic emission for a moving cracks has a measurable effect on the dynamics, introducing a selection criterion for the crack speed which, in the case of a strip geometry with fixed displacement at the boundaries, is missing in the continuum theory. The introduction of the efficiency $E(v)$ provides a way to predict the terminal crack speed in any loading configuration, at least given a preferred direction for crack propagation, and gives results comparable to experiments in the setup investigated. The departure from this prediction when a preferred direction is missing, is connected to microbranching which is responsible for the logarithmic scaling at marginal loadings. The well defined roughness exponent $\zeta \sim 0.45$ is connected in our simulations to the growth to macroscopic sizes of such branches and is possible only for higher loadings.

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