Ultrasound emission from advancing cracks in brittle materials

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Abstract

A new finite element model for simulating elasticity and fractures, is introduced and applied to the simplified case of two-dimensional samples. The phonon emission is analysed for simulations of planar cracks advancing at constant velocity. We show that the emission at the fracture surface can be interpreted in terms of dispersion relations for surface waves, consisting not only of the well known Raleigh and Love branch, but also from less known high frequency branches. The nature of these branches and the possibility of resonances is discussed.

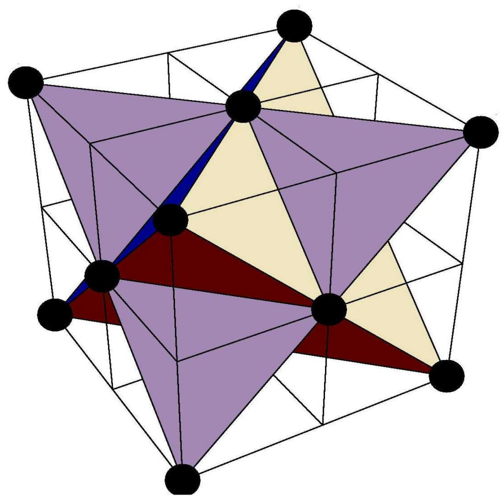
The model

Space is discretized using an fcc lattice and lattice points are connected using tetrahedral elastic finite elements. The Lagrangian of the system is given by:

$$L = \sum_{\alpha} \frac{1}{2} m \dot{\mathbf{u}}_{\alpha}^{2} - \sum_{\beta} \left\{ \frac{\lambda}{2} \sum_{i=1}^{3} \left[(\partial_i u_i)_{\beta} \right]^2 - \mu \sum_{i=1}^{3} \sum_{k=1}^{3} \left[\frac{1}{2} (\partial_i u_k + \partial_k u_i)_{\beta} \right]^2 \right\} \quad (1)$$

where:

- The α index spans all the lattice points
- The β index spans all the tetrahedral elements, and $(\partial_i u_j)_\beta$ is the unsymmetrized strain tensor at tetrahedron β



The unsymmetrized strain tensor is evaluated for each tetrahedron through:

$$\partial_i u_j = \alpha \sum_q v_i^q u_j^q \qquad \text{with} \qquad \alpha \sum_q v_i^q v_j^q = \delta_{ij}$$

and the vectors $\boldsymbol{v}_q$ join the center with the vertices of each tetrahedron

Fig. 1: unit cell for the fcc lattice. Tetrahedral elements are shown. Eight tetrahedra can be recognized, each connecting four lattice points.

Stress field of advancing cracks

A fracture consists in a set of *broken* tetrahedra: a tetrahedron is broken putting the elastic constants $\lambda = \mu = 0$ so that the contribution of the tetrahedron to the stress field is zero.

Reducing to one layer of cells we can consider two-dimensional samples. A fracture advancing at constant speed on a straight line is simulated applying a small displacement on the top and bottom faces of the sample, then breaking tetrahedra on a straight line, at a constant rate chosen to match the simulated crack speed. An analysis of the stress field reveals the presence of strong emission of *surface waves* on the crack surface.

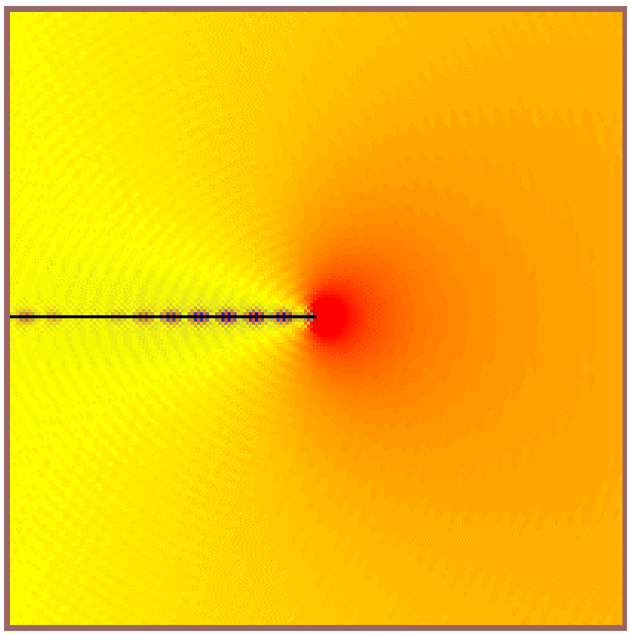


Fig. 2: The set-up for simulations. A crack is let to advance up to the middle of the sample. The stress field is then recovered.

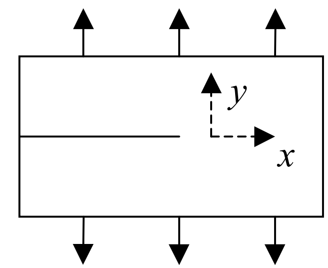
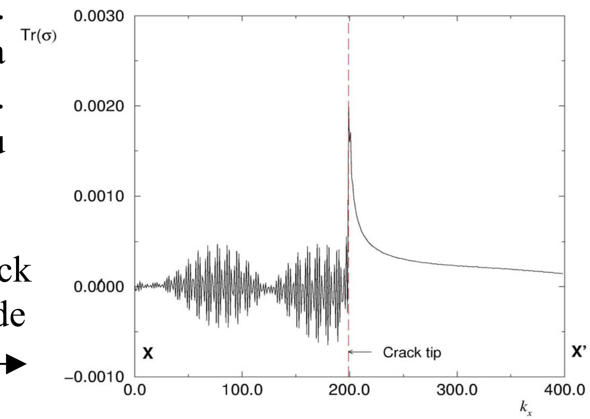


Fig. 3: Trace of the stress field for a crack advancing at half of the transverse waves speed. This is an enlargement from a sample of 400 x 400 tetrahedra. Intensity spans from blu (negative) to red (positive).

Fig. 4: Stress field along the crack direction, measured from one side of the sample to the other.



Description of the basic phenomenon

Let us consider a crack advancing at constant speed **v**. Due to the periodicity of the lattice, there is a periodic band structure. →

- In a system of reference co-moving with the crack tip, the frequency ω' is given by: $\omega' = \omega - \mathbf{k}\cdot\mathbf{v}$.
- In the co-moving frame, the crack tip is fixed, therefore $\omega' = 0$. For a crack advancing in the <100> direction this simplifies previous expression to $v = \omega / k_x$, where k_x covers higher Brillouin zones.
- Reducing to the first Brillouin zone, it can be written as: $$v = \frac{\omega}{(k'_x + g)}$$

  where **g** is a reciprocal lattice vector and k'_x belongs to the first Brillouin zone.

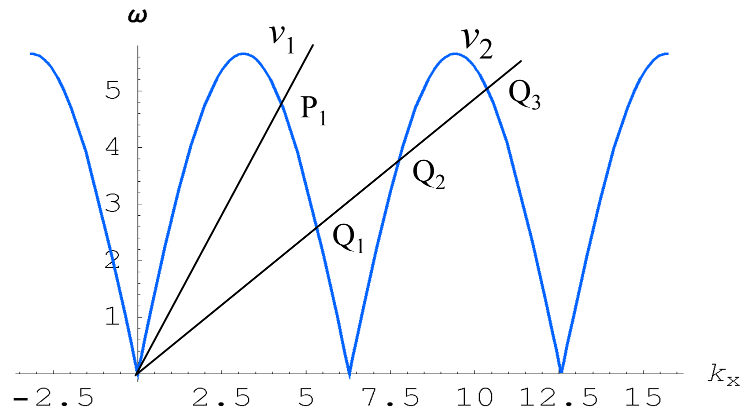


Fig. 5: A crack advancing at speed v_1 emits waves corresponding to the point P_1.
A crack advancing at speed v_2 emits waves corresponding to the points Q_1, Q_2 and Q_3; these last two points belonging to the higher Brillouin zone are not equivalent to Q_1.

- The presence of higher order Brillouin zones is nontrivial, due to the coupling between the advancing crack and surface waves.

Theoretical dispersion relations for surface waves

From eq. (1) the equations for dynamics can be recovered:

$$\rho \ddot{u}_{i,\alpha} = \frac{d}{dt}\frac{\partial L}{\partial \dot{u}_{i,\alpha}} = \frac{\partial L}{\partial u_{i,\alpha}}$$

which generates the equivalent of $\rho \ddot{\mathbf{u}} = \nabla \cdot \vec{\boldsymbol{\sigma}}$ for our tetrahedral finite element model.

Solutions of the equation give the dispersion relations for bulk waves in terms of **k** and ω. Surface waves can be found as linear combinations of bulk waves with the same ω, k_x and k_z, compelled on the surface by the boundary condition: $\vec{\boldsymbol{\sigma}} \cdot \hat{\mathbf{n}} = 0$

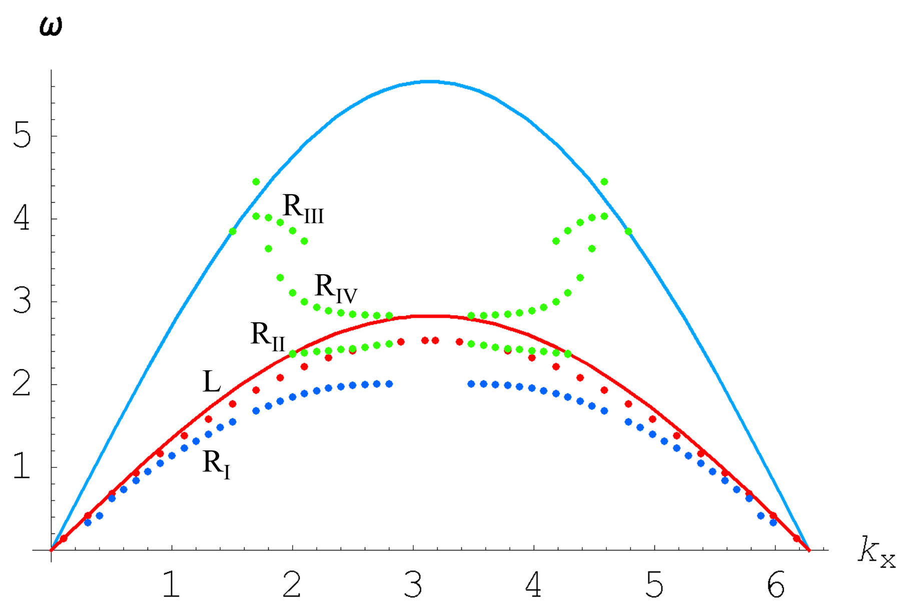


Fig. 6: Computed dispersion relations for surface waves in the first Brillouin zone are shown, for our tetrahedral finite element model. Colours have the following correspondence:

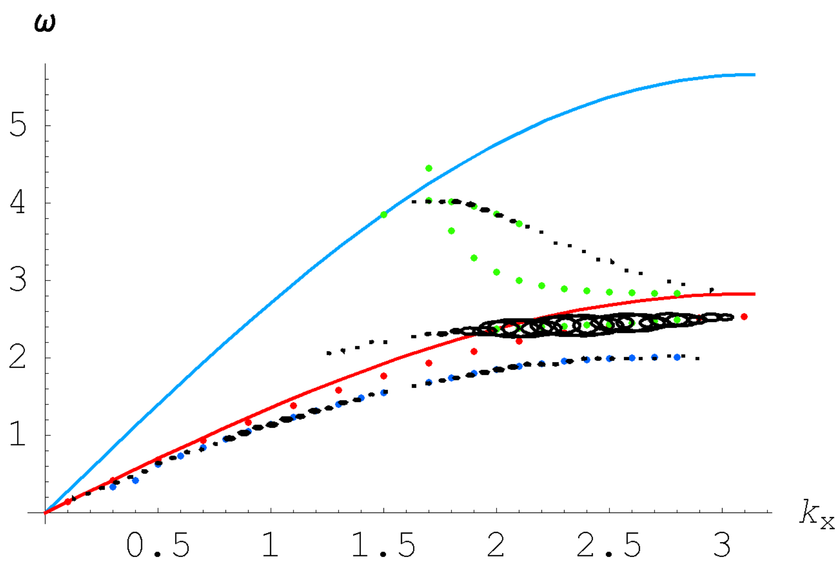
- $[R_I]$ Raleigh branch
- $[L]$ Love branch (polarized in the z-direction)
- $[R_{II\text{-}IV}]$ High-frequency branches

Lines show the dispersion relations for bulk modes. All the surface modes have $\mathrm{Im}(k_y) > 0$ to be damped for $y \to \infty$.

Direct measurement of surface waves

The dispersion relations can be measured exciting a free surface of an elastic sample. One k_x is excited and the frequencies of the corresponding modes are measured.

- Emissions lay on the theoretical branches.
- Emission are from the Raleigh and high-frequency branches R_{II} and R_{III}. No emission is observed from the R_{IV} branch.
- Measured branches show continuation of the theoretical branches. From the theory it can be shown that these continuation are modes having a complex ω. A weakening of the signal is in fact visible.



Note: the Love branch is not visible in these simulations due to its polarization

Fig 7: The dispersion relations measured on a free surface are reported and compared with the theoretical branches. More than one frequency can be excited for a given k_x. The size of circles reflects the intensity of the signals.

Emission from advancing cracks

Consider a snapshot at time t of the crack surface (see i.e. fig. 3).

- Fourier tranform of the surface profile gives peaks in the excited modes.
- Through $\omega = \mathbf{v} \cdot (\mathbf{k} + \mathbf{g})$ we can rebuild the dispersion relations.

Note that emission from the Love branch is not visible due to its polarization.

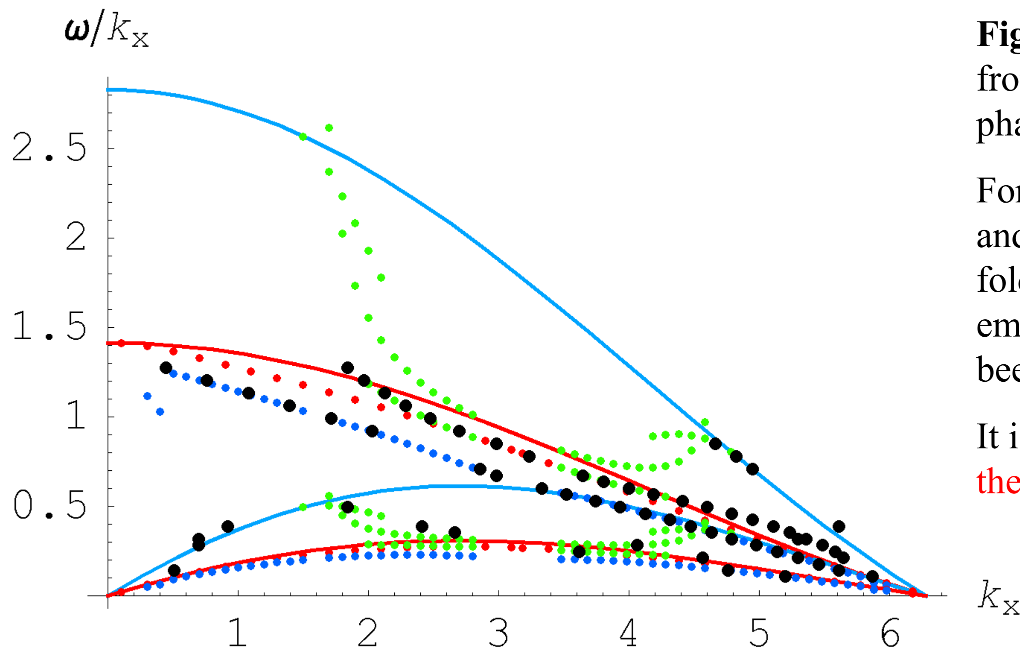


Fig. 8: Black points represent phonon emission from cracks advancing at constant speed. The phase velocity ω/k_x is plotted against k_x.

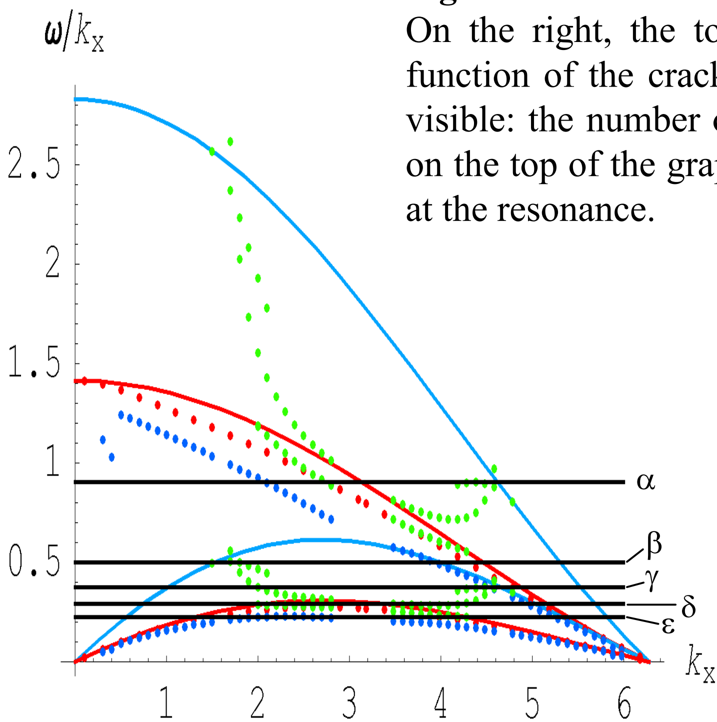
For simplicity, only emissions from the first and the second Brillouin zone are reported, and folded in the first Brillouin zone. However, emissions up to the fourth Brillouin zone have been observed.

It is evident that almost all the points lay on the theoretical branches or on their extensions.

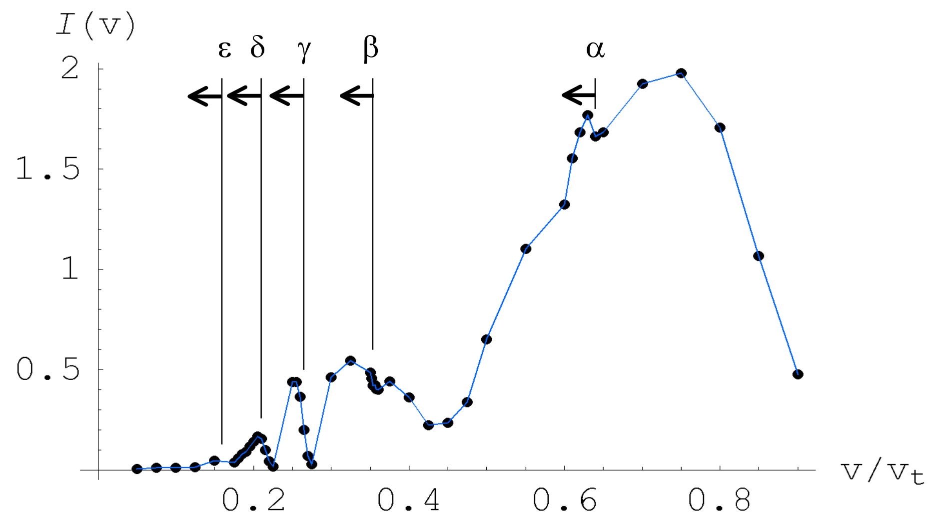
Searching for resonances

Possibility for resonances are when $\mathbf{v}_G = \partial\omega/\partial\mathbf{k} = \mathbf{v}$. For a crack is moving in the <100> direction: $v_G = \omega/k_x = v$

- Resonances are plateaux in ω/k_x: many modes couple to the advancing crack simultaneously.
- The Raleigh speed v_R is **no longer** the minimum speed for resonances.



Figs. 9-10: On the left, labelled lines show where resonances are expected to be seen. On the right, the total intensity I(v) of surface waves on the crack surface as a function of the crack speed is plotted. A systematic shift of the peaks to the left is visible: the number of excited modes increases in the direction of the arrows shown on the top of the graph. These correspond to the convexity of the dispersion relations at the resonance.



Conclusions

- A **new model** for simulating elasticity and fractures has been introduced.
- Simulations of planar fractures show **intense phonon emission** on the crack surface related to the dispersion relations of surface waves.
- Phonon emission is due not only to the Raleigh and Love branches, but also to less known **high frequency branches**. Emission is observed through the first and **higher Brillouin zones**.
- We show the possibility of **resonances** due to the structure of the dispersion relations, introducing **new critical values** for the crack speed besides the usual Raleigh and Yoffee critical speed values.

Bibliography

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